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Technical Notes

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Scaling of Trajectories of Elliptic Jets in Crossflow

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Nomenclature

d = circular jet diameter
d_h = hydraulic jet diameter
r = velocity ratio
x = downstream distance from the jet exit center
y = vertical distance above the jet exit center

I. Introduction

A jet in crossflow is a fundamental flow phenomenon that is important to a variety of engineering applications, such as aerodynamic flow control, film cooling of turbines and combustors, and jet-mixing enhancements, just to name a few. Over the past 60 years, numerous experimental and computational studies have been conducted on various aspects of the flowfield with much of the attention focused on the large-scale flow structure development, jet trajectories, scalar-mixing and transport properties, and other associated flow phenomena [1–27].

Although the scaling of the trajectories of a circular jet in crossflow has been studied for many years, there is still no generally accepted scaling parameter for the jet trajectory. The scaling parameters that have been proposed by researchers include d (by Kamotani and Greber [1], and Chassaing et al. [2]), rd (by Pratte and Baines [3]) and r^2d (by Keffer and Baines [4]), where r is the velocity ratio defined as jet velocity/crossflow velocity. Pratte and Baines [3], who derived the rd-scaling-based dimensional analysis, argued that the jet trajectory should not be normalized by d or r^2d, and went on to propose the power-law formulation

$$\frac{y}{rd} = A\left(\frac{x}{rd}\right)^B \tag{1}$$

where $A = 2.05$ and $B = 0.28$.

The preceding formulation is supported by a subsequent similarity analysis [6], and experimental results [13] at $r = 5$–25, which show that the trajectories are best scaled with rd, instead of d or r^2d. In fact, many of the existing experimental data can be converted to rd-scaling giving $1.2 < A < 2.6$ and $0.28 < B < 0.34$ [18,25]. More recently, Muppidi and Mahesh [24] plotted the jet trajectories using the published data in the literature, and found that they did not collapse onto a single curve when the coordinate axes were scaled by rd. They argued that the jet trajectory is dependent on, not only the velocity ratio and the jet diameter, but also the jet exit velocity profile and the crossflow boundary layer thickness. They went on to perform numerical calculations for various crossflow boundary layer conditions and jet velocity profiles, namely, parabolic and mean-turbulent, and from their numerical results, modify the existing power-law formulation to

$$\frac{y}{rd} = A'\left(\frac{x}{rd}\right)^{B'}\left(\frac{h}{d}\right)^{C'} \tag{2}$$

where A' and B' are constants, and $C' = 0.15$. The height up to which the jet is vertical, h, is dependent on the relative momentum of the jet and the crossflow boundary layer, and as proposed by the authors [24], can be obtained either analytically or using the y-coordinate of the trajectory at $x/d = 0.05$. Interestingly, a recent experimental study [26] on a circular jet in crossflow using parabolic or top-hat exit velocity profiles shows that rd-scaling gives a better collapse of the jet trajectories. Whether the poor collapse in the data using the scaling proposed by Muppidi and Mahesh [24] is due to imperfect parabolic and top-hat velocity profiles encountered in the experiment remains unclear.

To date, all the experimental and numerical studies on the scaling of transverse jets are confined to a circular jet only, and as far as we are aware, there is no corresponding scaling parameter for noncircular jets, due primarily to the lack of experimental data. In this technical note, we reanalyze our earlier experimental investigations [21,27] to see if there is an equivalent scaling parameter for elliptic jets in a crossflow (EJICF). The elliptic geometry is chosen for study because it is the simplest of the noncircular jet, and it also allows meaningful comparison with the result of a circular jet, which can be interpreted as an elliptic jet with an aspect ratio of one.

II. Experimental Setup

Experiments [21,27] were conducted in a recirculating water channel in the fluid mechanics laboratory at the National University of Singapore, and digital particle image velocimetry (DPIV) technique was used to obtain the velocity fields. Detailed description of the water channel, jet assemblies, and the DPIV technique can be found in [21,27] and will not be repeated here. A total of three jet assemblies were used, namely, two elliptic jets of aspect ratios two and three, and one circular jet, all with the same exit area. For each elliptic jet, two orientations were considered: one with the major axis aligned with the crossflow [referred to as a low aspect ratio (AR) jet], and the other with the minor axis aligned with the crossflow (referred to as a high aspect ratio jet). This combination allowed us to examine an equivalent of five jet geometries as shown in Table 1. In all cases, the crossflow in the water channel was set at 30 mm/s and the state of the boundary layer was found to be laminar, and followed the Blasius profile very closely. At the jet location, the boundary layer thickness as measured by a hot-film anemometer was approximately 25 mm. The velocity ratios considered ranged from one to five, and for each velocity ratio, a total of 1000 random sets of DPIV image pairs, with

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Table 1 Values for proposed power scaling law for jet geometries used in present study

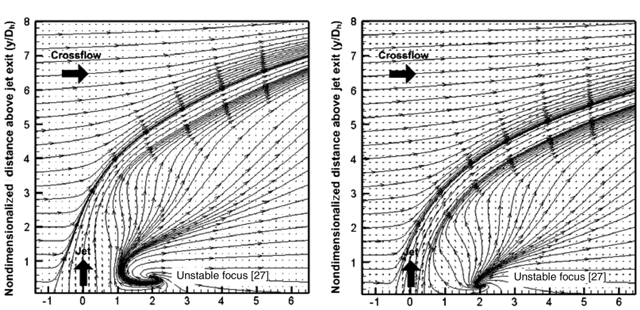
Configuration	AR	n	A^*	B^*
H, L	0.3	1.31	1.80	0.31
	0.5	1.09	1.55	0.28
	1.0	1.00	1.41	0.34
	2.0	1.16	1.15	0.33
	3.0	1.30	1.29	0.36

no relation to any phase of the flowfield, were captured to derive the time-averaged velocity fields.

III. Results and Discussions

Figure 1 shows typical time-averaged velocity fields for elliptic jets of AR = 0.5 and AR = 2.0, obtained by averaging 1000 random sets of instantaneous data [27]. From these results, jet center streamlines were identified and used as mean jet trajectories. The same definition was also used by Muppidi and Mahesh [24] in their numerical studies. Although other definitions, such as the locus of maximum velocity magnitude and locus of maximum concentration, have also been used by people, the final results did not seem to be significantly affected by the definition used. Figure 2 shows the mean elliptic jet trajectories obtained in the present study, and in-line with our earlier finding [27], low aspect ratio jets (i.e., AR = 0.3 and 0.5) have higher penetration into the crossflow than the high aspect ratio jets.

Because past studies have shown that rd-scaling gives the best collapse of circular jet trajectories, we decided to use the same scaling parameter for elliptic jets, but by replacing d with d_h, where d_h are the hydraulic diameters of the respective jet geometries. It is obvious from Fig. 3 that with the exception of a circular jet, rd_h-scaling does not give a good collapse of the jet trajectories.



a) AR = 0.5 b) AR = 2.0

Fig. 1 Mean velocity and streamline plots for elliptic jets in crossflow at $r = 3$.

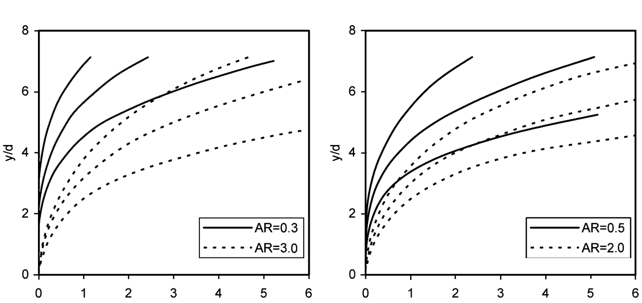
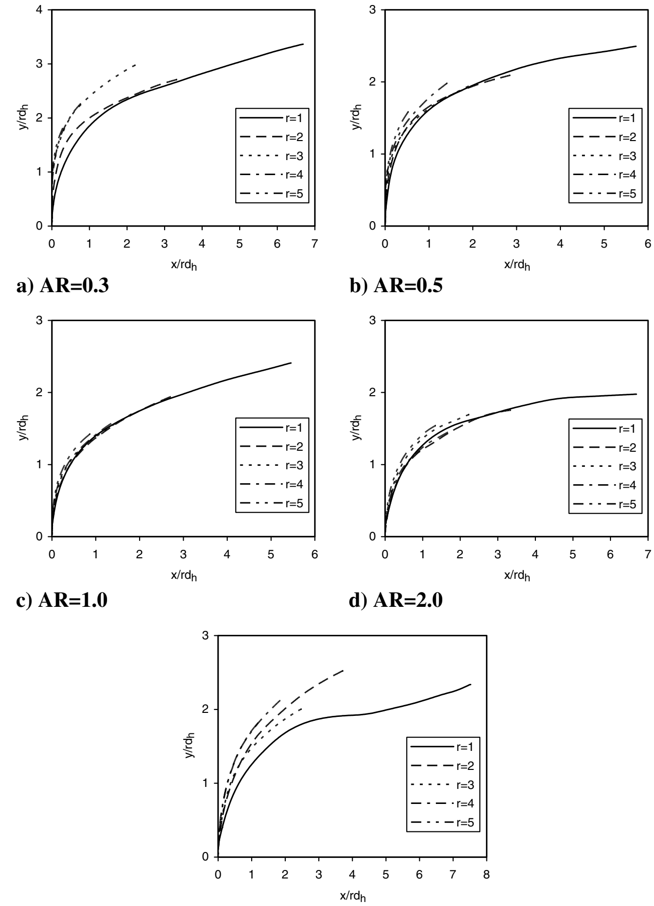


Fig. 2 Jet trajectories of low and high aspect ratio EJICF.



a) AR=0.3 b) AR=0.5

c) AR=1.0 d) AR=2.0

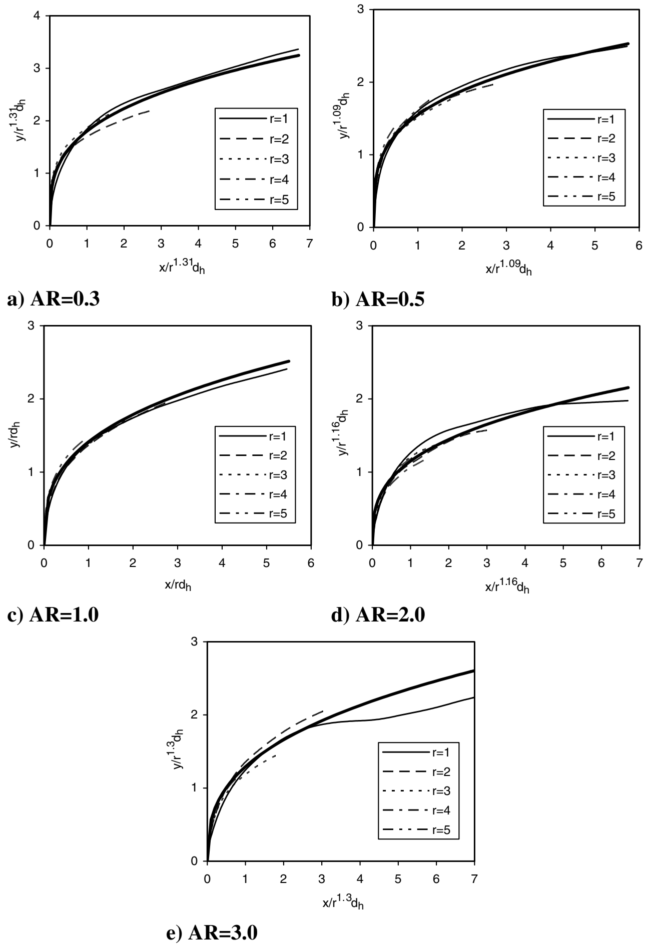
e) AR=3.0

Fig. 3 Scaling of the jet trajectories using rd_h.

Moreover, the collapse of the data deteriorates as the aspect ratio deviates further away from a circular jet (AR = 1.0). For example, jets with AR = 0.3 and 3.0 show a greater data scatter than that of AR = 0.5 and 2.0. These results evidently point to the inadequacies of rd_h-scaling for EJICF, and lead us to modify the scaling parameter to $r^n d_h$, with the corresponding power-law given by

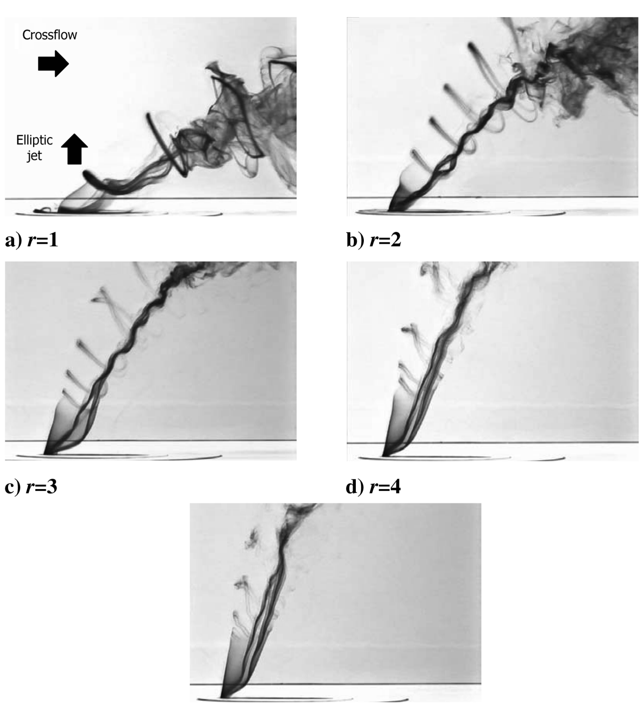
$$\frac{y}{r^n d_h} = A^* \left(\frac{x}{r^n d_h}\right)^{B^*} \tag{3}$$

where A^* and B^* are the coefficients which describe the power-law, and n is a constant which depends on the aspect ratio of the jet. Optimum n values are derived from curves of best fit using numerical regression, and A^* and B^* are determined from the jet trajectories collapsed using Eq. (3) and tabulated in Table 1. In the case of a circular jet, n is obviously equal to one. As clearly depicted in Fig. 4, although $r^n d_h$-scaling significantly improves the collapse of the elliptic jet trajectories, the extent of the collapse, once again, deteriorates as the elliptic jet geometry diverges further away from the circular shape. This finding is most interesting because it implies that within the "family" of curves in the same aspect ratio, having different velocity ratios (i.e., different exit jet velocities) significantly alters the characteristics of the jet trajectories. This suggests that the increased curvature of the leading-edge vortex loops coupled with the increase in their circulation (due to higher exit velocity), plays a decisive role in the jet trajectories. In fact, past studies on vortex filaments have shown that higher circulation and higher curvature would lead to higher induced velocity at the tip of the vortex loop. This is indeed evident in our flow visualization study, which shows that for the elliptic jet with the aspect ratio of 0.3, mutual interaction between neighboring vortex loops takes place when $r > 3.0$ (see Fig. 5). No such interaction is observed when $r < 2.0$ or when the jet geometry used is circular with an equivalent AR = 1.0 (see Fig. 6).



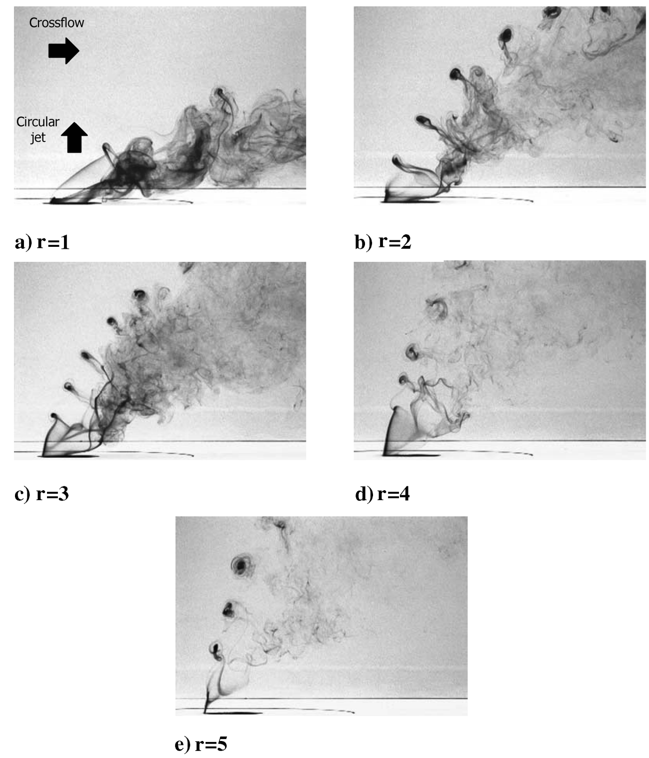
a) AR=0.3 b) AR=0.5 c) AR=1.0 d) AR=2.0 e) AR=3.0

Fig. 4 Collapse of the trajectories of elliptic jet in crossflow using $y/r^n d_h = A^*(x/r^n d_h)^{B^*}$.



a) r=1 b) r=2 c) r=3 d) r=4 e) r=5

Fig. 5 Flow visualization for AR = 0.3 elliptic jet in crossflow from r = 1–5.



a) r=1 b) r=2 c) r=3 d) r=4 e) r=5

Fig. 6 Flow visualization for AR = 1.0 circular jet in crossflow from r = 1–5.

In view of this unique behavior of elliptic jet structures, it may not be possible to collapse the elliptic jet trajectories in the same way as a circular jet, at least for the flow conditions investigated here.

No attempt was made to scale the elliptic jet trajectories using the method proposed by Muppidi and Mahesh [24] because, as pointed out by them, the technique is applicable to a circular jet only. Their comment is consistent with our experimental result which shows that for a fixed elliptic geometry, and a given jet exit velocity profile and boundary layer thickness, the jet trajectory with the major axis of the jet aligned with the crossflow (say, AR = 0.5) differs considerably from the case with the minor axis aligned with the crossflow (i.e., AR = 2.0). This finding suggests that, in addition to the jet velocity profile and the boundary layer thickness, jet exit geometry also plays an important role in determining the resultant jet trajectories.

IV. Conclusions

In this technical note, we have shown that although the trajectories of circular jets in crossflow can be scaled with rd, the unique flow dynamics of elliptic vortex loops suggest that it may not be possible to collapse the elliptic jet trajectories in the same way as a circular jet, at least for the range of flow conditions investigated here.

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